

The constraints of a straightforward soundness theorem (Theorem 1) lead to an abstract transition relation over this space:

$$((\llbracket (f \ e_1 \dots e_n)^\ell \rrbracket, \hat{\beta}, \widehat{ve}, \hat{t}), \equiv) \rightsquigarrow ((call, \hat{\beta}'', \widehat{ve}', \hat{t}'), \equiv'), \text{ where:}$$

$$\begin{aligned} \hat{d}_i &= \hat{\mathcal{E}}(e_i, \hat{\beta}, \widehat{ve}) \\ \hat{d}_0 &\ni (\llbracket (\lambda^{\ell'} (v_1 \dots v_n) \ call) \rrbracket, \hat{\beta}') \\ \hat{t}' &= \widehat{tick}(call, \hat{t}) \\ \hat{b}_i &= \widehat{alloc}(v_i, \hat{t}') \\ \hat{B} &= \{ \hat{b}_i : \hat{b}_i \in \widehat{Bind}_1 \} \\ \hat{\beta}'' &= (\hat{g}_{\hat{B}}^{-1} \hat{\beta}') [v_i \mapsto \hat{b}_i] \\ \widehat{ve}' &= (\hat{g}_{\hat{B}}^{-1} \widehat{ve}) \sqcup [\hat{b}_i \mapsto (\hat{g}_{\hat{B}}^{-1} \hat{d}_i)], \end{aligned}$$

and singleton bindings are reflexively equivalent:

$$\frac{\hat{b} \in \widehat{Bind}_1}{\hat{b} \equiv' \hat{b}},$$

and bindings between singletons are trivially equivalent:

$$\frac{\hat{\beta}(e_i) \in \widehat{Bind}_1 \quad \hat{b}_i \in \widehat{Bind}_1}{\hat{\beta}(e_i) \equiv' \hat{b}_i},$$

and untouched bindings retain their equivalence:

$$\frac{\hat{b} \equiv \hat{b}' \quad \hat{b} \notin \hat{B} \quad \hat{b}' \notin \hat{B}}{\hat{b} \equiv' \hat{b}'},$$

and bindings re-bound to themselves also retain their equivalence:

$$\frac{\hat{\beta}(e_i) \equiv \hat{b}_i}{\hat{\beta}(e_i) \equiv' \hat{b}_i}.$$

4.1 Solving the generalized environment problem

Under the direct product abstraction, the generalized environment theorem, which rules on the equality of individual bindings, follows naturally:

Theorem 3. *Given a compound abstract state $((call, \hat{\beta}, \widehat{ve}, \hat{t}), \equiv)$ and two abstract bindings, \hat{b} and \hat{b}' , if $\alpha^\eta(call, \beta, ve, t) \sqsubseteq ((call, \hat{\beta}, \widehat{ve}, \hat{t}), \equiv)$ and $\eta(b) = \hat{b}$ and $\eta(b') = \hat{b}'$ and $\hat{b} \equiv \hat{b}'$, then $ve(b) = ve(b')$.*

Proof. By the structure of the direct product abstraction α^η .